Comments on “Energy-efficient uplink multi-user MIMO”

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***Abstract*—In the paper “Energy-efficient uplink multi-user MIMO”, it was mentioned that the employed linear receiver does not change the variance of the elements of the noise vector. Then, the performance was calculated based on this proposition. In this note, we show that this proposition does not hold by providing the counter examples. In addition, we analyze how the performance will be affected by fixing this proposition.**

I. Counter examples

FOR consistency and compact explanation, we inherit all the symbols of the variables and the notations from [1]. In the equation (7) of [1], it was mentioned that the diagonal elements of $(\mathbf{U}^H\mathbf{U})^{-1}$ are identically equal to 1. The some parts of the subsequent analysis in [1] are based on this proposition. Thus, the validity of this proposition is not a minor issue. We found that this proposition does not hold. It is straightforward that the diagonal elements of $\mathbf{U}^H\mathbf{U}$ are identically equal to 1. However, this does not guarantee that the diagonal elements of $(\mathbf{U}^H\mathbf{U})^{-1}$ are identically equal to 1.

Just one counter example is sufficient to show this. For simplicity, consider the case when N=4, K=2, k_1=k_2=2. Assume that $\mathbf{H}_1$ and $\mathbf{H}_2$ are given as follows:

$$\mathbf{H}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (1)$$

$$\mathbf{H}_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}. \quad (2)$$

Then, through singular value decompositions, we have

$$\begin{aligned}\mathbf{H}_1 &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{aligned} \quad (3)$$

$$\begin{aligned}\mathbf{H}_2 &= \begin{pmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -1 & 0 & -1 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{2} & -\frac{1}{2} \\ \frac{-1}{\sqrt{2}} & 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 \\ 0 & -1 \\ \frac{-1}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{2}} & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \end{aligned} \quad (4)$$

From (3) and (4), $\dot{\mathbf{U}}_1$, $\dot{\mathbf{U}}_2$ and $\mathbf{U}$(refer to [1] for the definitions) are given as:

$$\dot{\mathbf{U}}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{pmatrix} \quad (5)$$

$$\dot{\mathbf{U}}_2 = \begin{pmatrix} 0 & 0 \\ 0 & -1 \\ \frac{-1}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{2}} & 0 \end{pmatrix} \quad (6)$$

$$\begin{aligned}\mathbf{U} &= [\dot{\mathbf{U}}_1 \dot{\mathbf{U}}_2] \\ &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & \frac{-1}{\sqrt{2}} & 0 \\ 0 & -1 & \frac{-1}{\sqrt{2}} & 0 \end{pmatrix}. \end{aligned} \quad (7)$$

From (7),

$$\mathbf{U}^H\mathbf{U} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1 & \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (8)$$

The diagonal elements of $\mathbf{U}^H\mathbf{U}$ correspond to the norms of each column of $\mathbf{U}$. As every column of $\mathbf{U}$ comes from the partial unitary matrices $\dot{\mathbf{U}}_1$ and $\dot{\mathbf{U}}_2$, it is trivial that all diagonal elements of $\mathbf{U}^H\mathbf{U}$ is equal to 1. Note that there are nonzero

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TABLE I

EXAMPLES OF $(\mathbf{U}^H\mathbf{U})^{-1}$ FOR RANDOMLY GENERATED $\mathbf{H}_1$ AND $\mathbf{H}_2$ WITH N=4, K=2, k_i=2.

Example	$(\mathbf{U}^H\mathbf{U})^{-1}$ for randomly generated $\mathbf{H}_1$ and $\mathbf{H}_2$			
1	**1.6017**	$-0.33+0.88i$	$0.92+0.46i$	$-0.30+0.83i$
	$-0.33-0.88i$	**2.70**	$0.50-1.77i$	$1.40-0.37i$
	$0.92-0.46i$	$0.50+1.77i$	**2.38**	$0.44+0.97i$
	$-0.30-0.83i$	$1.40+0.37i$	$0.44-0.97i$	**1.9161**
2	**1.27**	$-0.15+0.50i$	$-0.59+0.29i$	$-0.39-0.19i$
	$-0.15-0.50i$	**2.47**	$1.30+0.77i$	$-0.80+0.99i$
	$-0.59-0.29i$	$1.30-0.77i$	**2.03**	$-0.03+0.78i$
	$-0.39+0.19i$	$-0.80-0.99i$	$-0.03-0.78i$	**1.71**
3	**3.69**	$-1.89+0.70i$	$3.57-0.45i$	$0.98-0.33i$
	$-1.89-0.70i$	**2.80**	$-2.73-0.49i$	$-1.08-0.52i$
	$3.57+0.45i$	$-2.73+0.49i$	**4.90**	$1.25+0.02i$
	$0.98+0.33i$	$-1.08+0.52i$	$1.25-0.02i$	**1.59**

values in non-diagonal terms because the first column of $\dot{\mathbf{U}}_2$ is not orthogonal to the second column of $\dot{\mathbf{U}}_1$.

Owing to very simple form of $\mathbf{U}^H\mathbf{U}$ in (8), we can easily find its inverse, i.e., $(\mathbf{U}^H\mathbf{U})^{-1}$ as follows:

$$(\mathbf{U}^H\mathbf{U})^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & -\sqrt{2} & 0 \\ 0 & -\sqrt{2} & 2 & \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (9)$$

From (9), we know that the diagonal elements of $(\mathbf{U}^H\mathbf{U})^{-1}$ are not identical. The nonzero elements in the non-diagonal area of $\mathbf{U}^H\mathbf{U}$ make the diagonal elements of $(\mathbf{U}^H\mathbf{U})^{-1}$ be nonidentical. In order to further confirm this for the several general cases, $(\mathbf{U}^H\mathbf{U})^{-1}$ for arbitrarily generated complex-valued $\mathbf{H}_1$ and $\mathbf{H}_2$ are illustrated in Table I.

II. REMARKS ON THIS CORRECTION

From Table I, in addition to the fact that the diagonal elements are not identical, note that the diagonal elements are lager than 1. This implies that there exist noise enhancement if we use pseudo inverse of $\mathbf{U}$ as a linear detection matrix in the receiver (see the equations (4) and (5) in [1]). This is basically because the columns of $\mathbf{U}$ come from different unitary matrices and they are not fully mutually orthogonal.

Even if we fix this error, it does not affect the overall frame of the subsequent mathematical formulation in [1] because the received SNRs for each data stream are already expressed differently in the equation (8) of [1]. If we simply change the definition of λ_{ij}^2 in the equation (8) of [1] into $\lambda_{ij}^2/z_{(i-1)k_i+j}$ where $z_{(i-1)k_i+j}$ denotes the $((i-1)k_i+j)$th diagonal elements of $\left[(\mathbf{U}^H\mathbf{U})^{-1}\right]^H$, the remaining formulations in [1] do not need to be changed.

Despite this correction, some of the performance trends according to the system parameters analyzed in [1] may not be significantly changed because the term λ_{ij}^2 in the original(incorrect) expression for the achieved SNR η_{ik} in the equation (10) of [1] are already substantially uneven for different i and k regardless of the noise scaling correction factor $z_{(i-1)k_i+j}$, i.e., the diagonal elements of $\left[(\mathbf{U}^H\mathbf{U})^{-1}\right]^H$.

However, as the actual(correct) achieved SNR is smaller than η_{ik} used in [1] by a factor of $1/z_{(i-1)k_i+j}$, the performance results in [1] are optimistic compared to the correct ones and thus, they should be properly adjusted. In order to check the amount of noise variance scaling by this correction, we gather the diagonal elements of $\left[(\mathbf{U}^H\mathbf{U})^{-1}\right]^H$ for the randomly generated channel matrices $\mathbf{H}_1, \mathbf{H}_2, \cdots, \mathbf{H}_K$ whose elements are i.i.d zero mean complex Gaussian with unit variance and plot their empirical distributions in Fig. 1 and Fig. 2. For consistency, we consider the cases of the parameter set (N, K, k_i) used in [1]. In Fig. 1, as the number of users K increases, the noise enhancement gets more significant. In Fig. 2, as the number of user antennas k_i increases, the noise enhancement gets more significant. Note that as K or k_i increases, besides the increase of the average, the distribution gets wider and has longer tail. This implies that instantaneously the noise can be significantly enhanced and this situation more frequently happens as K or k_i increases.

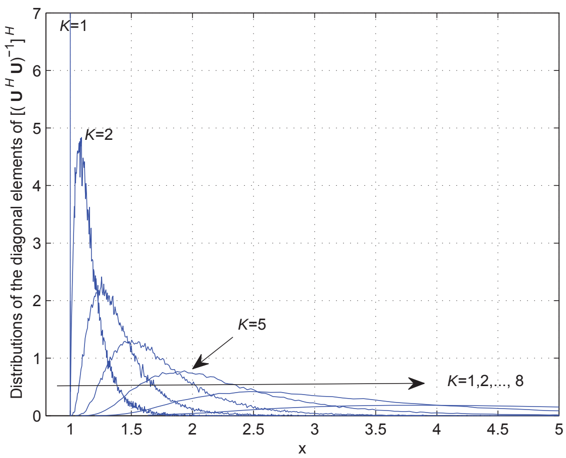

Fig. 1. Distributions of the diagonal elements of $\left[(\mathbf{U}^H\mathbf{U})^{-1}\right]^H$ according to K for the case with N=16 and k_i=2.

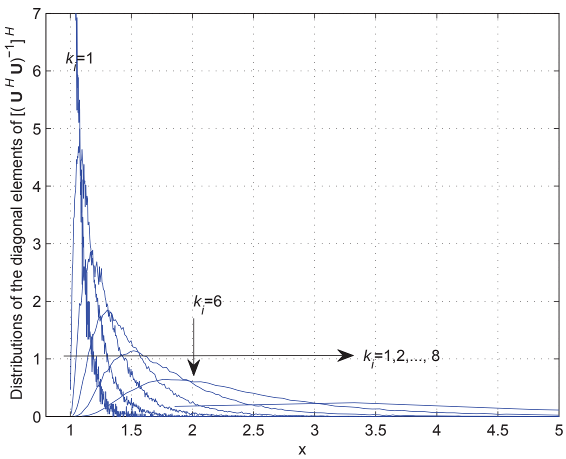

Fig. 2. Distributions of the diagonal elements of $\left[(\mathbf{U}^H\mathbf{U})^{-1}\right]^H$ according to k_i for the case with N=16 and K=2.

These observations conclude that the performance results in [1] should be underestimated than the provided values.

Especially for the systems with larger K or larger k_i, the performance should be adjusted more pessimistically.

References

[1] Guowang Miao, "Energy-efficient uplink multi-user MIMO," *IEEE Transactions on Wireless Communications*, vol. 12, no. 5, pp. 2302-2313, May 2013.